№371

$(x^2+y^2)^3=a^2x^2y^2$

Площадь найдём по формуле:

$$S=\iint_D pdpd\phi=\int_\alpha^B d\phi\int_{S_1}^{\alpha_2} pdp$$

Перейдём к полярным координатам:

$$\begin{cases} x=p\cos\phi \\ y=p\sin\phi \end{cases}$$

Получаем:

$$(p^2\cos^2\phi+p^2\sin^2\phi)^3=a^2p^2\cos^2\phi p^2\sin\phi$$

$$p^6=a^2p^4\cos^2\phi\sin^2\phi$$

$$p^2=a^2\cos^2\phi\sin^2\phi$$

$$p=a\cos\phi\sin\phi$$

$$S=\int_0^{\frac{\pi}{2}}d\phi\int_0^{a\cos\phi\sin\phi}pdp=\int_0^{\frac{\pi}{2}}\frac{p^2}{2}\Bigg|_0^{a\cos\phi\sin\phi}d\phi=\int_0^{\frac{\pi}{2}}a^2\cos^2\phi\sin^2\phi d\phi=\frac{a^2}{8}\int_0^{\frac{\pi}{2}}(1+\cos2\phi)(1-\cos2\phi)d\phi=$$

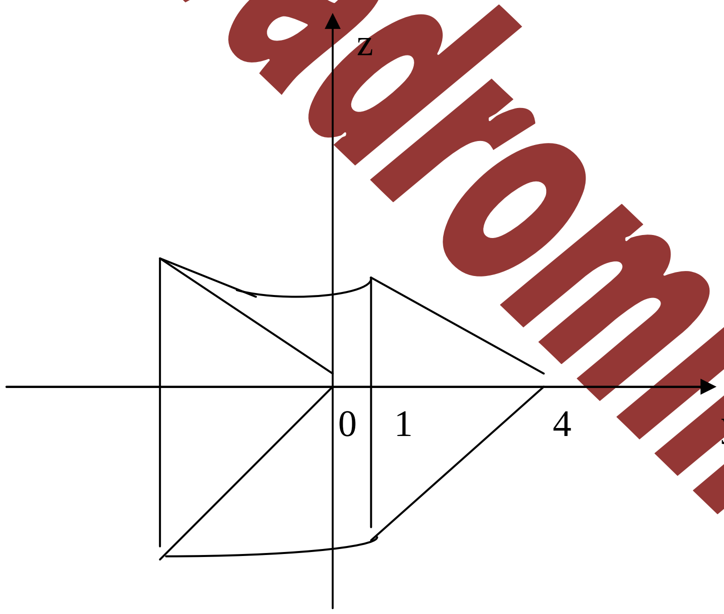
$$=\frac{a^2}{8}\int_0^{\frac{\pi}{2}}\left(1-\frac{1+\cos4\phi}{2}\right)d\phi=\frac{a^2}{16}\phi\Bigg|_0^{\frac{\pi}{2}}-\frac{a^2}{64}\sin4\phi\Bigg|_0^{\frac{\pi}{2}}=\frac{\pi a^2}{32}(кв.ед.).$$

№381

$z = 0, \quad z = x, \quad y = 0, \quad y = 4, \quad x = \sqrt{25 - y^2};$

Объём тела найдём по формуле:

$$V = \iiint_T dxdydz$$



$$V = \int_0^4 dy \int_0^{\sqrt{25-y^2}} dx \int_0^x dz = \int_0^4 dy \int_0^{\sqrt{25-y^2}} x dx = \int_0^4 \frac{x^2}{2}\Big|_0^{\sqrt{25-y^2}} dy = \frac{1}{2}\int_0^4 (25 - y^2)dy = \frac{1}{2}\left(25y - \frac{y^3}{3}\right)\Bigg|_0^4 = \frac{1}{2}\left(100 - \frac{64}{3}\right) \approx$$

$\approx 39{,}3(куб.ед.).$

№391

$$\int_L (x^2 - y)dx - (x - y^2)dy$$

L - дуга окружности:

$$\begin{cases} x = 5\cos t & A(5;0) \\ y = 5\sin t & B(0;5) \end{cases}$$

При $x = 5; \ y = 0 \quad t_1 = 0$

При $x = 0; \ y = 5 \quad t_2 = \frac{\pi}{2}$

$$\int_L p(x,y)dx + Q(x,y)dy = \int_{t_1}^{t_2} \{p(x(t), y(t)x'(t) + Q(x(t), y(t)y'(t)\}dt$$

Получаем:

$x'(t) = -5\sin t; \quad y'(t) = 5\cos t$

$$\int_L (x^2 - y)dx - (x - y^2)dy \int_0^{\frac{\pi}{2}} \{(25\cos^2 t - 5\sin t)(-5\sin t) + (25\sin^2 t - 5\cos t)5\cos t\}dt =$$

$$= \int_0^{\frac{\pi}{2}} (-125\cos^2 t\sin t + 25\sin^2 t + 125\sin^2 t\cos t - 25\cos^2 t)dt = 125\int_0^{\frac{\pi}{2}} \cos^2 t(d\cos t) + \frac{25}{2}\int_0^{\frac{\pi}{2}} (1 - \cos 2t)dt +$$

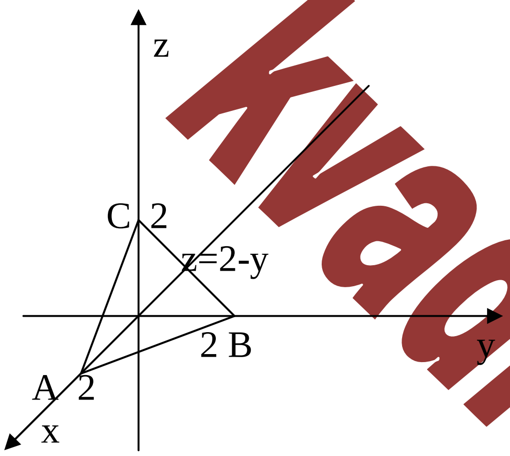
$$+125\int_0^{\frac{\pi}{2}} \sin^2 td\sin t - \frac{25}{2}\int_0^{\frac{\pi}{2}} (1 + \cos 2t)dt = \frac{125}{3} * \cos^3 t\Big|_0^{\frac{\pi}{2}} - 25\int_0^{\frac{\pi}{2}} \cos 2tdt + \frac{125}{3} * \sin^3 t\Big|_0^{\frac{\pi}{2}} =$$

$$= \frac{125}{3}\left(\cos^3\frac{\pi}{2} - \cos^3 0\right) - \frac{25}{2}\int_0^{\frac{\pi}{2}} \cos 2tdt(2t) + \frac{125}{3}\left(\sin^3\frac{\pi}{2} - \sin^3 0\right) =$$

$$= \frac{125}{3}(0^3 - 1^3) - \frac{25}{2}\sin 2t\Big|_0^{\frac{\pi}{2}} + \frac{125}{3}(1^3 - 0^3) = -\frac{125}{3} - \frac{25}{3}(\sin\pi - \sin 0) + \frac{125}{3} = -\frac{25}{2}(0 - 0) = 0.$$

№401

F=(x+z)i x+y+z-z=0



Плоскость:

$p = x + z \quad Q = 0 \quad R = 0$

$$1)\Pi = \iint Fnd\sigma = \iint_S (x+z)dydz + 0dzdx + 0dxdy = \iint_S (x+z)dydz = \iint_{Dyz} (x+z)dydz =$$

$$= \int_0^2 dy \int_0^{2-y} (2-y-z+z)dz = \int_0^2 dy \int_0^{2-y} (2-y)dz = \int_0^2 (2-y)z\Big|_0^{2-y} = \int_0^2 (2-y)(2-y-0)dy =$$

$$= \int_0^2 (2-y)^2 dy = \int_0^2 (4-4y+y^2)dy = \left(4y - \frac{4y^2}{2} + \frac{y^3}{3}\right)\Bigg|_0^2 = 4(2-0) - 2(2^2-0^2) + \frac{1}{3}(2^3-0^3) =$$

$$= 8 - 8 + \frac{8}{3} = \frac{8}{3};$$

$$2)Ц = \oint_L Fdr = \iint_\sigma nrotFd\sigma$$

$$rotF = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x+z & 0 & 0 \end{vmatrix} = 0 + 0 + j * \frac{\partial(x+z)}{\partial z} - k * \frac{\partial(x+z)}{\partial y} - 0 - 0 = 1i - 0 = 1i;$$

Формула Стокса:

$$Ц = \oint_L Fdr = \iint_\sigma 1dydz + 0dzdx + 0dxdy = \iint_\sigma dydz = \iint_{D_{yz}} dydz = \int_0^2 dy \int_0^{2-y} dz = \int_0^2 z\Big|_0^{2-y} dy =$$

$$= \int_0^2 (2-y-0)dy = \left(2y - \frac{y^2}{2}\right)\Bigg|_0^2 = 2(2-0) - \frac{2^2-0^2}{2} = 4 - 2 = 2;$$

Формула Остроградского:

$$\Pi = \iint Fnd\sigma = \iiint_V divFdV$$

$$divF = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z} = (x+y)'_x + 0'_y + 0'_z = 1;$$

http://kvadromir.com/arutunov_sbornik_9.html — решебник Арутюнова Ю.С. Контрольная работа 9. Вариант 1. Номера 371, 381, 391, 401, 411

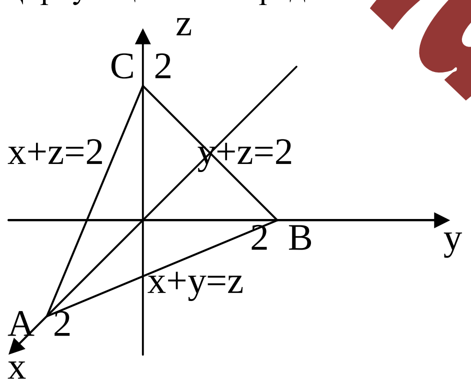
$$П = \int_0^2 \int_0^{2-x} \int_0^{2-x-y} 1dxdydz = \int_0^2 dx \int_0^{2-x} dyz\Big|_0^{2-x-y} = \int_0^2 dx \int_0^{2-x} (2-x-y-0)dy = \int_0^2 dx\left((2-x)y - \frac{y^2}{2}\right)\Bigg|_0^{2-x} =$$

$$= \int_0^2 dx\left((2-x)(2-x-0) - \frac{1}{2}((2-x)^2 - 0^2)\right) = \int_0^2 \left((2-x)^2 - \frac{1}{2}(2-x)^2\right)dx =$$

$$= \frac{1}{2}\int_0^2 (2-x)^2 dx = \frac{1}{2}\int_0^2 (4-4x+x^2)dx = \frac{1}{2}\left(4x - \frac{4x^2}{2} + \frac{x^3}{3}\right)\Bigg|_0^2 =$$

$$= \frac{1}{2}\left(4(2-0) - 2(2^2-0^2) + \frac{1}{3}(2^3+0^3)\right) = \frac{1}{2}\left(8-8+\frac{8}{3}\right) = \frac{4}{3}.$$

Циркуляция непосредственно:



$$Ц = \oint_L \overline{F}dr = \int_{AB} (x+z)dx + \int_{BC} 0dy + \int_{CD} 0dz = \int_0^2 (x+0)dx = \frac{x^2}{2}\Bigg|_0^2 = \frac{2^2-0^2}{2} = 2;$$

Поток непосредственно

$$П = \iint_S (P\cos\alpha + Q\cos\beta + R\cos\gamma)dS = \iint_S (x+z)\frac{1dx}{\sqrt{3}dy} = \frac{1}{\sqrt{3}}\int_0^2 dx \int_0^{2-x}(x-0)dy =$$

$$= \frac{1}{\sqrt{3}}\int_0^2 xy\Big|_0^{2-x} dx = \frac{1}{\sqrt{3}}\int_0^2 x(2-x-0)dx = \frac{1}{\sqrt{3}}\int_0^2 (2x-x^2)dx = \frac{1}{\sqrt{3}}\left(\frac{2x^2}{2} - \frac{x^3}{3}\right)\Bigg|_0^2 =$$

$$= \frac{1}{\sqrt{3}}\left(2^2 - 0^2 - \frac{1}{3}(2^3-0^3)\right) = \frac{1}{\sqrt{3}}\left(4 - \frac{8}{3}\right) = \frac{1}{\sqrt{3}} * \frac{4}{3} = \frac{4}{3\sqrt{3}} = \frac{4\sqrt{3}}{9}.$$

№411

$$\overline{F} = (6x + 7yz)\overline{i} + (6y + 7xz)\overline{j} + (6z + 7xy)\overline{k}$$

$$p = 6x + 7yz; \quad Q = 6y + 7z; \quad R = 6z + 7y;$$

$$rot\overline{F} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6x + 7yz & 6y + 7z & 6z + 7y \end{vmatrix} = \overline{i}\left(\frac{\partial(6z + 7xy)}{\partial y} - \frac{\partial(6y + 7xz)}{\partial z}\right) - \overline{j}\left(\frac{\partial(6z + 7y)}{\partial x} - \frac{\partial(6x + 7yz)}{\partial z}\right) +$$

$$+ \overline{k}\left(\frac{\partial(6y + 7xz)}{\partial x} - \frac{\partial(6x + 7yz)}{\partial y}\right) = \overline{i}(7x - 7x) - \overline{j}(7y - 7y) + \overline{k}(7z - 7z) = 0$$

Следовательно, поле потенциальное:

$$div\overline{F} = \frac{\partial p}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z};$$

$$\frac{\partial p}{\partial x} = 6; \quad \frac{\partial Q}{\partial y} = 6; \quad \frac{\partial R}{\partial z} = 6;$$

$$div\overline{F} = 6 + 6 + 6 = 18 \neq 0$$

Следовательно, данное поле не соленоидальное. Найдём потенциальное поле $\overline{F}$.

$$u(x, y, z) = \int_{x_0}^{x} P(x, y_0, z_0)dx + \int_{y_0}^{y} Q(x, y, z_0)dy + \int_{z_0}^{z} R(x, y, z)dz + c.$$

Таким образом:

$$u(x, y, z) = \int_0^x 0dx + \int_0^y 6ydy + \int_0^z (6z + 7xy)dz + c = \frac{6y^2}{2} + \frac{6z^2}{2} + 7xyz = 3y^2 + 3z^2 + 7xyz$$

Здесь в качестве начальной точки взята точка (0;0;0).