№289

$$a)\int \frac{\sin x dx}{\sqrt[3]{3+2\cos x}} = -\int \frac{d\cos}{\sqrt[3]{3+2\cos x}} = -\frac{1}{2}\int (3+2\cos x)^{-\frac{1}{3}} d(2\cos x) = -\frac{1}{2}\sqrt[3]{(3+2\cos x)^2}\,\frac{3}{2} + C =$$

$$= -\frac{3}{4}\sqrt[3]{(3+2\cos x^2)} + C.$$

Проверка:

$$\left(-\frac{3}{4}\sqrt[3]{(3+2\cos x)^2} + C\right)' = -\frac{3}{4} * \frac{1}{2}\frac{1}{\sqrt[3]{3+2\cos x}}(-2\sin x) = \frac{\sin x}{\sqrt[3]{3+2\cos x}};$$

$$б)\int x^2 \sin 4x dx = \left[\begin{matrix} u = x^2; \quad du = 2xdx \\ dv = \sin 4xdx \\ v = -\frac{1}{4}\cos 4x \end{matrix}\right] = -\frac{x^2}{4}\cos 4x + \int \frac{1}{4}\cos 4 * 2xdx = -\frac{x^2}{4}\cos 4x + \frac{1}{2}\int c * \cos 3xdx =$$

$$= \left[\begin{matrix} u = x; \quad du = dx \\ dv = \cos 4xdx \\ v = \frac{1}{4}\sin 4x \end{matrix}\right] = -\frac{x^2}{4}\cos 4x + \frac{1}{2}\left(\frac{x}{4}\sin 4x - \frac{1}{4}\int \sin 4xdx\right) = -\frac{x^2}{4}\cos 4x + \frac{x}{8}\sin 4x + \frac{1}{32}\cos 4x + C$$

Проверка:

$$\left(-\frac{x^2}{4}\cos 4x + \frac{x}{8}\sin 4x + \frac{1}{32}\cos 4x + C\right)' = -\frac{2x}{4}\cos 4x + \frac{x^2}{4}\sin 4x + \frac{\sin 4x}{8} + \frac{x}{8} * 4\cos 4x - \frac{4}{32}\sin 4x =$$

$$= x^2 \sin 4x;$$

$$в)\int \frac{(x^2 - x + 1)}{x^4 + 2x^2 - 3}dx$$

$$x^4 + 2x^2 - 3 = 0$$

$$D = 4 + 12 = 16$$

$$x_1^2 = \frac{-2-4}{2} = -3; \quad x_2^2 = \frac{-2+4}{2} = 1$$

$$\int \frac{(x^2 - x + 1)}{x^4 + 2x^2 - 3}dx = \int \frac{x^2 - x + 1}{(x^2+3)(x-1)(x+1)} = I$$

$$\frac{x^2 - x + 1}{(x^2+3)(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C_x + D}{x^2+3} = \frac{(Ax+A)(x^2+3) + (Bx-B)(x^2+3) + (Cx+D)(x^2-1)}{(x^2+3)(x-1)(x+1)}$$

$$\begin{array}{l|l} x^3 & A + B + C = 0 \\ x^2 & A - B + D = 1 \\ x^1 & 3A + 3B - C = -1 \\ x^0 & 3A - 3B - D = 1 \end{array}$$

$$A = -B - C$$

$$B = A + D - 1 = -B - C + D - 1$$

$$B = \frac{-C + D - 1}{2}$$

$$C = 3A + 3B + 1 = -3B - 3C + 3B + 1 = -3C + 1 \qquad C = \frac{1}{4}$$

$$D = 3A - 3B - 1 = -3B - 3C - 3B - 1 = -6B - \frac{3}{4} - 1 = -6.$$

$$\frac{-C + D - 1}{2} - \frac{7}{4} = \frac{3}{4} - \frac{3D}{4} + 3 - \frac{7}{4}$$

$$4D=2\Rightarrow D=\frac{1}{2};\qquad B=\frac{-\frac{1}{4}+\frac{1}{2}-1}{2}=-\frac{3}{8}$$

$$A=\frac{3}{8}-\frac{1}{4}=\frac{3-2}{8}=\frac{1}{8}$$

$$I=\frac{1}{8}\int\frac{dx}{x-1}-\frac{3}{8}\int\frac{dx}{x+1}+\int\frac{\frac{1}{4}x+\frac{1}{2}}{x^2+3}dx=\frac{1}{8}\ln(x-1)-\frac{3}{8}\ln(x+1)+\frac{1}{8}\int\frac{dx^2}{x^2+3}+\frac{1}{2\sqrt{3}}\int d\frac{x\sqrt{3}}{\left(\frac{x}{\sqrt{3}}\right)^2+1}=$$

$$=\frac{1}{8}\ln(x-1)-\frac{3}{8}\ln(x+1)+\frac{1}{8}\ln(x^2+3)+\frac{1}{2\sqrt{3}}arctg\frac{x}{\sqrt{3}}+C$$

г) $$\int\frac{\left(\sqrt{x}-1\right)\left(\sqrt[6]{x}+1\right)}{\sqrt[3]{x^2}}dx=$$

$$\left|\begin{aligned}&t=\sqrt[6]{x}\qquad t^6=x\\&dx=\left(t^6\right)'dt=6t^5dt\\&t^2=\left(\sqrt[6]{x}\right)^2=\sqrt[3]{x}\\&t^3=\left(\sqrt[6]{x}\right)^3=\sqrt{x}\end{aligned}\right.$$

$$=\int\frac{(t^3-1)(t+1)}{(t^2)^2}*6t^5dt=6\int(t^3-1)(t+1)*tdt=6\int(t^5+t^4-t^2-t)dt=6\left(\frac{t^6}{6}+\frac{t^5}{5}-\frac{t^3}{3}-\frac{t^2}{2}\right)+C=$$

$$=6\left(\frac{x}{6}+\frac{\sqrt[6]{x^5}}{5}-\frac{\sqrt{x}}{3}-\frac{\sqrt[3]{x}}{2}\right)+C=x+\frac{6}{5}\sqrt[6]{x^5}-\frac{1}{2}\sqrt{x}-3\sqrt[3]{x}+C.$$

№299

$$\int_{-2}^{8}\sqrt{x^3+8}dx$$

Нужно определить значение подынтегральной функции для следующих значений аргумента $(h=1)$: $x_0=-2; x_1=-1; x_2=0; x_3=1; x_4=2; x_5=3; x_6=4; x_7=5; x_8=6; x_9=7; x_{10}=8.$

Находим соответствующие значения $f(x)=\sqrt{x^3+8}$

$y_0=\sqrt{-8+8}=0;\ y_1=\sqrt{-1+8}=\sqrt{7}=2{,}646;\ y_2=\sqrt{8}=2{,}828;\ y_3=\sqrt{1+8}=3;\ y_4=\sqrt{8+8}=4;$
$y_5=\sqrt{27+8}=5{,}916;\ y_6=\sqrt{72}=8{,}485;\ y_7=\sqrt{133}=11{,}533;\ y_8=\sqrt{224}=14{,}967;\ y_9=\sqrt{351}=18{,}735;$
$y_{10}=\sqrt{520}=22{,}804.$

По формуле Симпсона находим

$$\int_{-2}^{8}\sqrt{x^3+8}=\frac{1}{3}(y_0+y_{10}+4(y_1+y_3+y_5+y_7+y_9)+2(y_2+y_4+y_6+y_8))=$$

$$=\frac{1}{3}(0+22{,}804+4(2{,}646+3+5{,}916+11{,}533+18{,}735)+2(2{,}828+4+8{,}485+14{,}967))=$$

$$=\frac{1}{3}(22{,}804+167{,}32+60{,}56)\approx 83{,}561.$$

№309

Вычислить несобственный интеграл или доказать его расходимость.

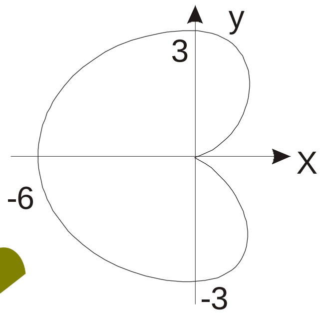
$$\int_0^4 \frac{dx}{\sqrt[3]{(x-3)^2}} = \int_0^3 \frac{dx}{\sqrt[3]{(x-3)^2}} + \int_3^4 \frac{dx}{\sqrt[3]{(x-3)^2}} = \lim_{b\to 3}\int_0^b \frac{dx}{\sqrt[2]{(x-3)^2}} + \lim_{a\to 3}\int_a^4 \frac{dx}{\sqrt[2]{(x-3)^2}} =$$

$$= \lim_{b\to 3}\left(3\times(x-3)^{\frac{1}{3}}\Big|_0^b \right) + \lim_{a\to 3}\left(3\times(x-3)^{\frac{1}{3}}\Big|_a^4 \right) = \lim_{b\to 3}\left(3\sqrt[3]{b-3} - 3\sqrt[3]{0-3}\right) + \lim_{a\to 3}\left(3\sqrt[3]{4-3} - 3\sqrt[3]{a-3}\right) =$$

$$\left(3\times 0 + 3\sqrt[3]{3}\right) + \left(3\times 1 - 3\times 0\right) = 3 + 3\sqrt[3]{3} \approx 7.327.$$

№319

Вычислить длину кардиоиды $r = 3(1-\cos\varphi)$.



$L = \int_{\varphi_1}^{\varphi_2} \sqrt{r^2 + (r')^2}\, d\varphi$. В данном случае $\varphi_1 = 0, \quad \varphi_2 = 2\pi,$

$r = 3(1-\cos\varphi), \quad r' = 3\sin\varphi.$

Учитывая симметричность кардиоиды относительно оси Ох запишем:

$$L = 2\int_0^{\pi} \sqrt{9(1-\cos\varphi)^2 + 9\sin^2\varphi}\quad d\varphi = 6\int_0^{\pi}\sqrt{1 - 2\cos\varphi + \cos^2\varphi + \sin^2\varphi}\; d\varphi =$$

$$6\sqrt{2}\int_0^{\pi}\sqrt{1-\cos\varphi}\; d\varphi = \left| \begin{array}{c} tg\frac{\varphi}{2} = t; \quad d\varphi = \frac{2dt}{1+t^2} \\ \cos\varphi = \frac{1-t^2}{1+t^2}; \quad \varphi = 0 \Rightarrow t = 0 \\ \varphi = \pi \Rightarrow t = \infty \end{array} \right| = 6\sqrt{2}\int_0^{\infty}\sqrt{\frac{1+t^2-1+t^2}{1+t^2}} \times \frac{2dt}{1+t^2} =$$

$$6\sqrt{2}\lim_{b\to\infty}\int_0^b \left(1+t^2\right)^{-\frac{3}{2}} \times 2\sqrt{2}\; t\; dt = 12\lim_{b\to\infty}\int_0^b \left(1+t^2\right)^{-\frac{3}{2}} d\left(1+t^2\right) = 12\lim_{b\to\infty}\frac{-2}{\sqrt{1+t^2}}\Bigg|_0^b = 12\lim_{b\to\infty}\left(\frac{-2}{\sqrt{1+b^2}} - \frac{-2}{1}\right) =$$

$$12(0+2) = 24.$$