№283

$$а)\int \frac{x^3dx}{\sqrt{1-x^8}}=\frac{1}{4}\int\frac{dx^4}{\sqrt{1-x^8}}=[x^4=t]=\frac{1}{4}\int\frac{dt}{\sqrt{1-t^2}}=\frac{1}{4}\arcsin t+C=\frac{1}{4}\arcsin(x^4)+C.$$

Проверка:

$$\left(\frac{1}{4}\arcsin(x^4)+C\right)'=\frac{1}{4}\frac{1}{\sqrt{1-x^8}}*4x^3=\frac{x^3}{\sqrt{1-x^8}}$$

$$б)\int x3^x dx=\begin{bmatrix} u=x, & du=dx \\ dV=3^x dx & \\ V=\frac{3^x}{\ln 3} & \end{bmatrix}=\frac{x3^x}{\ln 3}-\int\frac{3^x}{\ln 3}dx=\frac{x3^x}{\ln 3}-\frac{3^x}{\ln^2 x}+C.$$

Проверка:

$$\left(\frac{x3^x}{\ln 3}-\frac{3^x}{\ln^2 3}+C\right)'=\frac{x3^x\ln 3+3^x}{\ln 3}-\frac{3^x\ln 3}{\ln^2 3}=x3^x$$

$$в)\int\frac{|3x-7|dx}{x^3+4x^2+4x+16}=I$$

$$\frac{3x-7}{x^3+4x^2+4x+16}=\frac{A}{x+4}+\frac{Bx+C}{x^2+4}=\frac{A(x^2+4)+(x+4)(Bx+C)}{(x+4)(x^2+4)}$$

$$\begin{array}{l|l} x^2 & A+B=0 \\ x^1 & 4B+C=3 \\ x^0 & 4A+4C=-7 \end{array}\quad\Rightarrow\quad \begin{array}{l} A=-B \\ C=3-4B \\ -4B+12-16B=-7 \end{array}$$

$$20B=19$$

$$B=\frac{19}{20};A=-\frac{19}{20};C=3-\frac{19}{20}=-\frac{4}{5}$$

$$I=-\frac{19}{20}\int\frac{dx}{x+4}+\frac{1}{20}\int\frac{19x-16}{x^2+4}dx=-\frac{19}{20}\ln|x+4|+\frac{19}{40}\int\frac{d|x^2+4|}{x^2+4}-\frac{4}{5}\int\frac{dx}{x^2+4}=-\frac{19}{20}\ln|x+4|+$$
$$+\frac{19}{40}\ln|x^2+4|-\frac{2}{5}arctg\frac{x}{2}+C.$$

$$г)\int\frac{dx}{\sqrt{x+3}+\sqrt[3]{(x+3)^2}}=\begin{bmatrix} x+3=t^6 \\ dx=6t^5dt \end{bmatrix}=\int\frac{6t^5dt}{t^3+t^4}=6\int\frac{t^2dt}{1+t}=6\int(t-1)dt+6\int\frac{dt}{1+t}=$$
$$=\frac{6t^2}{2}-6t+6\ln|1+t|+C=3\sqrt[3]{x+3}-6\sqrt[6]{x+3}+6\ln\left|1+\sqrt[6]{x+3}\right|+C.$$

№293

$$\int_{-3}^{7}\sqrt{x^3+32}dx=I$$

$h=1$

$y_0=2{,}236;\ y_1=4{,}899;\ y_2=5{,}568;\ y_4=5{,}657;\ y_4=5{,}749;\ y_5=6{,}325;\ y_6=7{,}681;\ y_7=9{,}798;\ y_8=12{,}53;$
$y_9=15{,}748;\ y_{10}=19{,}365.$

$$I=\frac{1}{3}\left(y_0+y_{10}+4(y_1+y_3+y_5+y_7+y_9)+2(y_2+y_4+y_6+y_8)\right)=$$

$$=\frac{1}{3}(2{,}236+19{,}365+4(4{,}899+5{,}657+6{,}325+9{,}798+15{,}748)+2(5{,}568+5{,}745+7{,}681+12{,}53))=$$

$$=\frac{1}{3}(21{,}601+169{,}708+63{,}048)=84{,}786.$$

№303

$$\int_{-1}^{+\infty}\frac{dx}{x^2+x+1}=\lim_{b\to+\infty}\int_{-1}^{b}\frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}=\lim_{b\to+\infty}\left(\frac{2}{\sqrt{3}}arctg\frac{x+\frac{1}{2}}{\sqrt{3}}*2\right)\Bigg|_{-1}^{b}=$$

$$=\frac{2}{\sqrt{3}}\lim_{b\to+\infty}\left(arctg\frac{2b+1}{\sqrt{3}}-arctg\left(-\frac{1}{\sqrt{3}}\right)\right)=$$

$$=\frac{2}{\sqrt{3}}\left(\frac{\pi}{2}+\frac{\pi}{6}\right)=\frac{2}{\sqrt{3}}*\frac{4\pi}{6}=\frac{4\pi}{3\sqrt{3}}.$$

Несобственный интеграл сходится.

№313

$$r=3(1+\cos\phi)$$

$$S=\frac{1}{2}\int_{\alpha}^{\beta}r^2d\phi$$

$$S=\frac{1}{2}*\int_0^{2\pi}(3(1+\cos\varphi))^2d\varphi=\frac{1}{2}\int_0^{2\pi}9(1+\cos\varphi)^2d\varphi=\frac{9}{2}\int_0^{2\pi}(1+2\cos\varphi+\cos^2\varphi)d\varphi=$$

$$=\frac{9}{2}2\int_0^{\pi}(1+\cos\varphi+\cos^2\varphi)d\varphi=9\int_0^{\pi}(1+\cos\varphi)d\varphi+9\int_0^{\pi}\cos^2\varphi d\varphi=9(\varphi+\sin\varphi)\Big|_0^{\pi}+9\int_0^{\pi}\frac{1+\cos2\varphi}{2}d\varphi=$$

$$=9(\pi+\sin\pi-0-\sin0)+\frac{9}{2}\varphi\Big|_0^{\pi}+\frac{9}{4}\sin2\varphi\Big|_0^{\pi}=9\pi+\frac{9}{2}\pi=\frac{27\pi}{2}.$$